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Mass-mobilization with noisy conditional beliefs ☆

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Abstract

We examine the role played by information in shaping popular expectations in the process of political mobilization, and the development of revolutionary movements in particular. The vast majority of the people face the trade-off between the 'risky' option of joining the revolutionary movement and the 'safe' option of supporting the present regime. The large majority of the people are influenced by the expectations of a successful campaign of either political side. Only if the expected or perceived support for the opposition exceeds a critical mass, does the revolution become effective. We show that the growth-dynamics of revolutionary movements heavily depends on the parameters determining the perception of the actual state of support. And, we will argue that it is the informational aspect that provides the necessary bias to get out of the self-confirming outsider position. Especially, as we argue, it provides an explanation for the symbolic violence used by terrorist groups as it serves them as an instrument by which the mobilization dynamics is turned into a format where information has the largest impact. Surprisingly, this will in general be the situation where a majority of the people dissents from the acts of terror.

Keywords: Collective action; Mass-mobilization; Revolution; Dynamical system

JEL classification: C15; C71

1. Introduction

We follow Schelling (1978) and Granovetter (1978) in assuming that the typical revolutionary group's opening dilemma can be illustrated by an 'S'-shaped response curve. This approach is

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a threshold model in which there are typically three different equilibria: a low stable equilibrium (the hard core), an intermediate unstable equilibrium and a high stable equilibrium. This model reflects an early mobilization problem of revolutionary groups: until it has established an effective base of support, it is unlikely to survive, but as long as it hasn't arrived at a threshold initializing a bandwagon effect, it will be unlikely to establish a surviving base of support.

Kuran (1989, 1991a,b) suggests that the early mobilization problem will only be resolved through a catalytic event This event will serve as a spark that will result in a dramatic change of support for the revolutionaries and catapult it beyond the threshold. This spark, he notes, is often difficult to discern, explaining why revolutionary outbreaks seem to 'appear out of nowhere'.

Lohmann (1993, 1994a,b) observes that much political action (and in particular, demonstrations) is a means of signaling: most individuals are ignorant of the true state of the world (e.g., how many people support a revolutionary group), and demonstrations serve to deliver this information. The present paper offers an explanation of sparks by an explicit analysis of the dynamic adjustment process from one state of the world to another. We think of the adjustment process as the result of two subsequent steps: (1) formation of beliefs about the true state of the world — the *information component*, and (2) a bandwagon effect depending on the belief — the preference component.

(1) In the first step people perceive the true state of the world. This perception significantly depends on the level of information which forms their belief. In case of full information the believed state of the world is identical to the true state of the world. However, with decreasing quality of information, distortion effects are likely to occur in the sense that deviations from the true state as the believed state of the world may be expected.
(2) The behavioral component specifies how agents respond to a (believed) state of the world. We define the *response function* which represents the cumulative responsiveness and measures the willingness to join the group subject to the (believed) share of the people who have already made the decision to sign on.

2. A dynamic model of mass-mobilization

A characteristic of revolutions is the small time-scale at which they occur. Some of them, like the French Revolution, take everybody by surprise, leaving political scientists and historians with the question of how to explain these unforeseen events. Kuran (1989) lists a couple of reasons that could serve as a powerful cocktail, triggering the developments as they in fact occur. Reasons leading a revolutionary movement to overthrow the establishment may be due to effects induced as part of a strategic choice on either side; others appear more or less coincidental. Though the literature uses arguments that would ask for a dynamical framework, these dynamics are never made explicit. This paper can be considered as a generalization of the dynamics that are first introduced by Granovetter (1978) and further treated by McCormick and Owen (1996). First we will describe these *deterministic* dynamics, which will give food for thought and lead us to adapt the model that would best fit the description of a discrete time/continuous state Markov process.

The present article discusses the process of mass-mobilization as resulting from a *threshold function*, i.e., a function $f:[0,1]\rightarrow[0,1]$ that relates each fraction of the people that is believed to support the opposition to the fraction of the people that will join on the basis of this information. Below we will call the fraction of people that, at time t, stands firm for the opposition and gives actual support, the *true state* of the world at t. Then the related deterministic dynamics of the true states can be formulated in terms of the evolution of the system

$$\begin{aligned} x_o &= \delta \\ x_{t+1} &= f(x_t) \end{aligned} \tag{1}$$

where δ is taken as the initial true state. As is suggested by Granovetter (1978), Kuran (1989) and McCormick and Owen (1996), the mapping f is typically an increasing and continuous mapping with an S-shaped graph. Below we will interpret its shape as the way the masses are spread over the political spectrum represented by the interval [0,1], where 0 means showing openly complete loyalty to the opposition (revolutionaries) and 1 stands for being extremely loyal to the group representing the public power, i.e., regime or government. The *S*-shape is motivated as follows. Joining a revolutionary movement when it is small is a risky and hazardous undertaking. If it is not believed that there are sufficiently many individuals joining to be successful in establishing the goals of the opposition, the majority of people would rather refrain from giving support to the party that is most likely going to lose. On the other hand, if the revolutionaries have a majority support, regime adepts may choose to support the revolution as a second best option. This models the situation that most people have populist preferences, which lead them to join those that they believe to be leading, even if not the most preferred party.

Note that we are dealing with a universe Ω of all citizens, or at least of all citizens who can actually contribute in some way to the insurgency. N, the number of these, is of the order of 10,000 or larger (most likely, of the order of several million). Now, each individual, on the basis of his beliefs (subjective probabilities) and preferences (utility function) will decide whether to join the insurgency.

Essentially, individual i assigns utilities to the four cases

$e1$ = Insurgency wins, i does not join
$e2$ = Insurgency wins, i joins
$e3$ = Insurgency loses, i does not join
$e4$ = Insurgency loses, i joins

We can expect that i would rather be on the winning side, and so $u(e2)>u(e1)$, and $u(e3)>u(e4)$.

Suppose, now, that this individual believes that a fraction x of the population supports the insurgency. He then assigns probability $\pi(x)$ – which is an increasing function of x – to the event: Insurgency wins. Now, the individual will join the insurgency if the expected utility of doing so will exceed the utility of not joining, i.e., if

$$\pi(x)u(e2) + (1-\pi(x))u(e4) > \pi(x)u(e1) + (1-\pi(x))u(e3)$$

Some algebra reduces this condition to

$$\pi(x) > \frac{u(e3)-u(e4)}{u(e2)-u(e1)+u(e3)-u(e4)}$$

Now π is a strictly increasing function of x. Thus, the value of x, call it ζ, which makes the two sides of this last inequality equal is a threshold for the individual: the individual will join the insurgency if $x>\zeta$. Since the utilities are individual and the subjective probabilities, π, are individual, this ζ is an individual threshold, $\zeta(i)$.

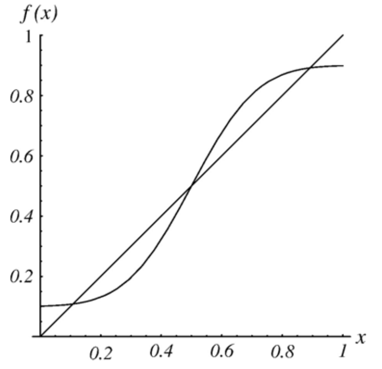

Fig. 1. An S-shaped reaction curve.

These values $\zeta(i)$ will now give us the threshold function f, i.e.

$$f(x) = \#\{i \in \Omega | \zeta(i) < x\}/N$$

A typical S-shaped threshold function is depicted in Fig. 1.

Observe that the graph of the function f intersects the graph of the identity function on [0,1].[1] Such points are restpoints for the dynamics as, for each such point x, the relation $x=f(x)$ holds. In the above figure there are three of them, say $x_L < x_M < x_H$. The fraction x_L stands for the equilibrium fraction of the people who would constitute the small and stable part of the people that remains as a support if the initial state is (chosen to be) smaller than x_M. If $x_O > x_M$ then the true state of the world rapidly converges towards x_H, the scenario at which revolution is most likely to occur, as the opposition may count on large support. Needless to say, though x_M may be a restpoint, it is an unstable one and will generically not occur.

We differ in our approach from the present literature in that we distinguish between two different aspects of the mass-mobilization process: (1) First, at time t people develop a perception of the true state of the world $b(x_t)$, the *believed state* conditional on the true state at t, x_t, and (2) society reacts to $b(x_t)$ with $r(b(x_t))$, the fraction of people that would give support to the opposition if it is commonly believed that a fraction $b(x_t)$ is doing so. In particular, this means that the relevant dynamics of mass-mobilization are summarized by the composite mapping $r \circ b$. The deterministic model is contained in this general version of dynamics with f as reaction function and $b(x)=x$ for all x; people are reacting to the true state as if it were known. In our model, however, beliefs about the true state are explicitly modeled as we will argue that full information about the true state of the world is impossible to achieve. Besides that, Kuran (1989) mentions already *information* about the true state of the world as one of the factors having significant impact on the evolutionary processes in politics. Lohmann (1993, 1994a,b) also points out the important role that information (or, for that matter, *dis*information) plays in such processes.

A lack of information may be due to different causes, which we will classify as *exogenous* and *endogenous* factors. The exogenous factors find their origin in the problem of knowing the actual state of the world due to physical as well as mental constraints; the perception of the true state is

[1] In particular the continuity of f together with Brouwer's Fixed Point Theorem guarantees the existence of points of intersection.

no more than the aggregate of individual mental experiences, and none has full access to the others' thoughts. In such case the best knowledge consists of an estimate of revolutionary activity only, as for example can be derived via interviews and polls in the (independent) media.

The endogenous factors leading to incomplete information are due to the roles of the two competing parties. For instance, as far as clandestine operating revolutionary groups are concerned, the informational impact is substantial due to their very nature. The repressive force of the regime will make it impossible for revolutionary groups to operate in the open and highlight their state of being, which underlines the fact that it is this information that finds itself at the very heart of the game that is played between the adversaries. Below we will show how distortion of information about the true state may well form part of a propaganda campaign on either side as a combination of effects is shown to shape the people's preferences, which then consequently may sharpen the bandwagon-effect.

Thus, for both type of reasons, individuals in society are able to retrieve partial information only and therefore imperfect competence for detecting the true level of membership resides.[2] This deformation of knowledge about the true level of support for revolution is structural for all, even for the members of a revolutionary movement as an open declaration of support for a clandestine group may involve high risks of getting arrested. Despite the impossibility of knowing the true state of support we will assume that people form beliefs about the true state instead. Then this is exactly the informational aspect that we will model by the component b. For instance, one might think of b as taking a sample from the distribution induced by the true state. Then component r represents the behavioral part of the process and describes the bandwagon effect subject to $b(x_t)$.

3. Research questions

This study will try to shed light on the role of both components in the mobilization dynamics, and, in particular, the interplay between noisy beliefs of states and the people's reaction to that. We will aim at answering the following questions, if possible.

1. Does information affect the dynamic process? If so, how sensitive are these results with respect to the level of information? Does information change the stability of fixed points of the threshold function that are used as predicted end-states by earlier mentioned literature? How stable are these equilibria under the stochastic dynamics?
2. The spread of the believed state around the true state depends on exogenous as well as endogenous factors. The exogenous part is explained by physical constraints to know the actual state of the world. The endogenous factors are determined by the adversaries, each of which could try to influence public opinion by the use of (distorted) information. Then can we say something about the impact of information as part of a propaganda campaign. If it is possible to shape peoples preferences through information, how could it be deployed as an instrument to counter the influence of terrorist activities?
3. The paradox of terrorism is that each attack seems to underline that the public opinion is against them. Does our model provide insights as to why it could be profitable for terrorists to use such strategy?

[2] From an informational aspect this is the problem considered by Condorcet (1785).

Remarkc. What is actually described here is a discrete time continuous state Markov process. Can we learn something from the theory of this kind of systems?

4. The role of information on the system dynamics

4.1. Going from real state to believed state

Above we argued that the assumption of full information seems far too stringent for our purposes. Moreover, though the corresponding dynamics

$$x_{t+1} = f(x_t) \tag{2}$$

may be used to motivate the evolutionary pressure towards one of the stable equilibria, it does not serve our goal in explaining why it is possible that revolts actually occur as it would only endorse the salient point that marks the beginning of a revolutionary group.

Suppose that at a certain moment the real state is $x \in [0,1]$. We model the perceived or *believed* state as a random variable $B(x)$ with values in [0, 1]. Hence we model the dynamics as if society is represented by an agent whose random belief is distributed around the true state of the world[3]. As a first approach we make the following assumptions:

1. Believed states are conditional upon the true state x.
2. The closer a state is to the true state x, the more likely this will be perceived as the true state[4]. This idea is implemented by our choice of probability density functions for $B(x)$, which all attain maximal values around x.
3. We take the standard deviation τ of $B(x)$ as a measure of information, in the sense that decreasing τ reshapes the density function in favor of knowing the true value of x.

From here there are as many ways to proceed as the number of probability distributions that would fit our desiderata. For analytical convenience we will restrict ourselves to a family of cumulative probability functions that we will subsequently refer to as *polynomial*. This family is rich enough to capture the main features of our model and, as it turns out, our main findings may be replicated using other intuitive families of distribution. Fig. 2 below shows the typical shape of the probability density function (PDF) and cumulative density function (CDF) for this class of distributions.

[3] A good question is as to why society can be represented by a single agent, i.e., do individuals all receive the same signal? It might be more realistic to think of each individual i's signal as consisting of the sum of two components, $w+v_i$, where w is common to all society's members, and v_i is a personal signal; these v_i could then be independent. In that case, it is the social component w that gives rise to variability in the outcome: since N (the size of our population) is considered large, then, contingent on w, it would be possible to give relatively small confidence intervals for the fraction of the population that elects to join the insurgency. If, for example, we have $N=40{,}000$, and our calculations tell us that, for a given w, the expected fraction is 0.437, then we can have 95% confidence that the actual number of individuals is between 0.432 N and 0.442 N.

[4] Admittedly, this might not be the case. For example, we might consider the case of a regime that controls means of communication, and is of course interested in making the insurgency look as weak as possible. In that case, the believed state $B(x)$ would probably be considerably for smaller than x. We will simply point out that, in such a case, the calculations/simulations would be carried out essentially as they are in our present paper, though of course the results would be different (we would find that the insurgency is much less likely to succeed). We reserve this for a later paper.

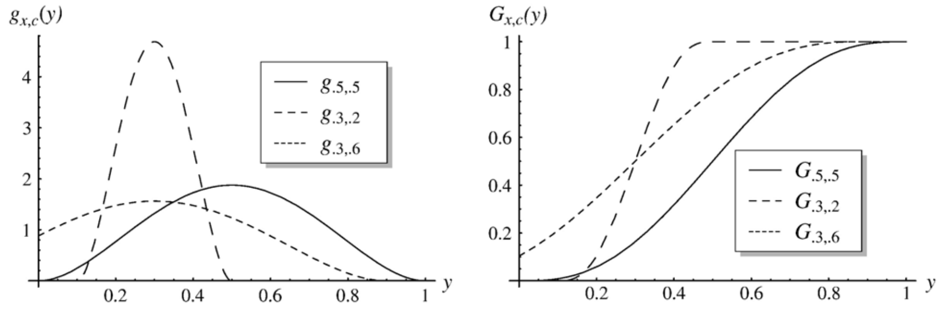

Fig. 2. Examples of $g_{x,c}$ and $G_{x,c}$ for different combinations of (x,c).

4.2. Intermezzo: the family of polynomial distributions

Here we will introduce the distribution corresponding to $B(x)$, the belief about the true state conditional on x. Given $x \in [0,1]$ and constant $c > 0$, define the function $g_{x,c}$ by

$$g_{x,c}(y) = \frac{15(x^2 - c^2 - 2xy + y^2)^2}{16c^5} \text{ if } y \in [x - c,\ x + c], \tag{3}$$

0 otherwise.

Then $g_{x,c}$ defines a probability density function on [0,1] so long as $[x-c, x+c] \subset [0,1]$ since it is easy to verify that, in such case

$$\int_0^1 g_{x,c}(y)\mathrm{d}y = 1$$

By symmetry x is the mean of the distribution. Below we will assume that $g_{x,c}$ describes the distribution of the believed state $B(x)$. Then its variance is given by

$$\mathrm{Var}\ (B(x)) = E[(B(x) - x)^2] = \int_0^1 (y - x)^2 g_{x,c}(y)\mathrm{d}y = c^2/7 \tag{4}$$

So we may interpret c as a measure of the information about the true state as the standard deviation is given, for such combinations of x and c, by $c/\sqrt{7}$. As c goes to zero, all the probability mass is centered at x which in the limit is equivalent to truly knowing x.

For values of x and c such that $x - c < 0$ or $x + c > 1$ we may still use the function $g_{x,c}$ in order to construct a cumulative probability density function by adding point masses in an appropriate way. In general, $x,c \in [0,1]$ we define the cumulative probability density function $G_{x,c}$:$[0,1] \to [0,1]$ by

$$G_{x,c}(y) = \begin{cases} \int_{x-c}^{y} g_{x,c}(s)\mathrm{d}s & \text{if } 0 \leq y < 1 \\ 1 & \text{if } y = 1 \end{cases} \tag{5}$$

Fig. 2 shows the graphs of the functions $g_{.5,.5}$,$g_{.3,.2}$, and $g_{.3,.6}$ together with corresponding cumulative density functions. The mappings $g_{.5,.5}$ and $g_{.3,.2}$ are the PDF's corresponding to $G_{.5,.5}$ and $G_{.3,.2.}$ Note that according to $G_{.3,.6}$ there is a fraction $G_{.3,.6}(0)$ – approx. 0.10 – of people that denies the existence of people in support of the revolution. In case $x < c$ the mean of the corresponding distribution is larger than x; similarly it is less than x if $x>1-c$. For example the mean of the distribution corresponding to CDF $G_{.3,.6}$ is approximately $0.31>.3$.

4.3. Going from believed state to reaction on believed state

At this point, we will assume that the threshold or reaction function has an S-shaped graph, just as sketched in Fig. 1. However, in order to focus on information effects only the model is treated where the reaction function has a graph that is point-symmetric in (0.5, 0.5). In particular, we use the following r_σ:

$$r_\sigma(x)=\alpha+(1-2\alpha)\Phi\{0.5,\sigma\}(x), \text{ for } x\in[0,1].$$

Here $\Phi\{.5, \sigma\}$ is the cumulative distribution function (CDF) corresponding to the normal distribution with mean .5 and standard deviation σ. For small σ, the number α represents the fraction of the people that can be considered *hardcore* revolutionaries as they will join the opposition, irrespective of the actual support. Notice that, by symmetry considerations, there is also a fraction α of people that would always be in favor of the established regime. In all the examples below the value $\alpha=0.1$ is assumed. Note, as in the second graph in Fig. 3, an increase in σ leads to a slightly increased fraction of hardcore membership.

The steepness of the curve around 0.5 indicates how many people are – more or less-opportunists. The heap of mass in the middle of the spectrum indicates that there are plenty of people who will just join any party, just as long as they believe it will be successful in achieving the majority. This is illustrated in Fig. 3 which shows the graphs of r_σ for six different values of σ. An increase in σ goes hand in hand with an increasing spread of the people over the political spectrum — at least in terms of willingness to support the revolutionary movement. To stress the difference in the spread of the political spectrum and that of the information we will frequently use σ_r as parameter included in the definition of r_σ.

5. Experimental set-up

5.1. Results

By running simulations we create non-linear time-series from which we may learn two things:

1. The long-term distributions of the states of the system. In the deterministic case the system will lock in, as (generically) the system will converge to one of the stable equilibria; which

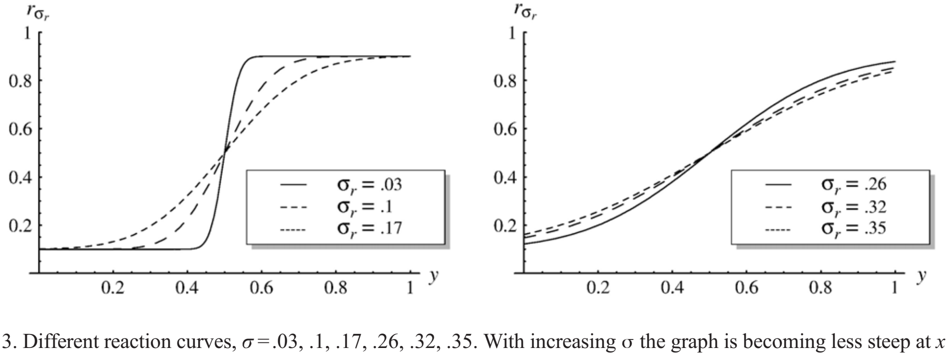

Fig. 3. Different reaction curves, $\sigma=.03, .1, .17, .26, .32, .35$. With increasing σ the graph is becoming less steep at $x=.5$.

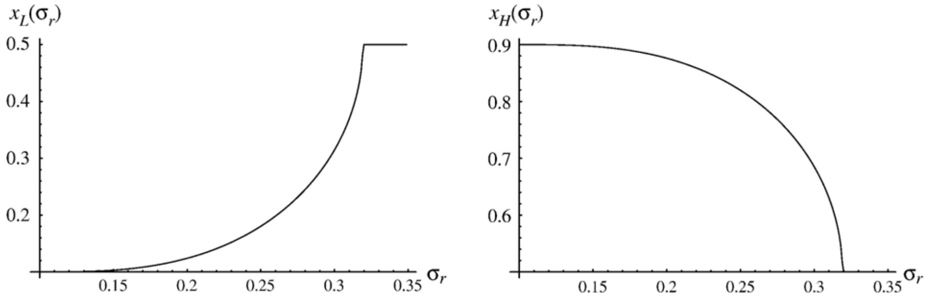

Fig. 4. Calculated equilibria as function of σ.

equilibrium survives is only determined by the initial state. The way we include uncertainty about the believed state allows the system to jump out of the deterministically stable states. Then the question is whether these restpoints may still be seen as restpoints for the dynamics, i.e. whether the system returns arbitrarily often to sufficiently nearby points.

2. The time it would take the system to jump from the status quo, which is taken to be about the left lower equilibrium, to a state exceeding state $x=.5$. This would be the time that a revolutionary movement would need to get assured of the support of at least half of the population. In this respect we adhere to the approach of Kuran (1989, p54) who calls 'a shift in collective sentiment a *revolution* if it exceeds .5 units'. In this manner we are able to distinguish between processes yielding the same long-term distributions of the states, but which not are equally volatile. The issue of time-scale is just as relevant in this study as it is in evolution theory: where it is a stylized fact in politics that one regime will be followed by another, there is less to say about the speed and the frequency of transitions. Long-time distributions may not provide the desired insight into the likeliness that a revolt will occur in a certain period-jumping times will.

Note, however, that our system dynamics show all characteristics of a discrete-time continuous-state Markov process[5] which guarantees the existence of limiting long term distributions, both of the states of the system and of the jumping times. Nevertheless, these are theoretically given and in general hard to calculate precisely. Given the type of distributions we use, a better idea is to use simulation techniques and obtain approximate results. We will show that even with a relatively small number of rounds of simulations one is able to notice clear-cut differences in the characteristics of the dynamics, which was set out as our goal.

We ran the simulations in *Mathematica*. First a pseudo-random generator is used to draw from a uniform distribution on [0, 1] and subsequently the obtained random numbers are transformed in such a way that the combined process yields the desired probability distribution. For details, see Appendix A.

5.2. Long-run distribution of states

Below we comment on the results for several runs of the above algorithm, each with different combination of c and σ_r. Each run consists of 10^5 iterations, the results of which are graphically presented in Appendix B.

[5] See, e.g., Gikhman and Skorokhod (1969).

In order to detect the role of information on the system properties, we compare the results obtained from the simulations with the qualitative properties of the deterministic dynamics (1) with $f = r_\sigma$. We will treat the latter more extensively before discussing our main findings.

5.3. Steady states of the deterministic system

First we give a qualitative description for the deterministic dynamics in terms of the equilibrium states. Fig. 3 shows that for low values of σ_r there are three restpoints for the dynamics, a low (stable) equilibrium state $x_L(\sigma_r)$, a high (stable) equilibrium state $x_H(\sigma_r)$ and an unstable restpoint $x_M(\sigma_r)=.5$. For values of σ_r around .32 the *S*-shape almost coincides with the 45° line.

Fig. 10 shows that for $\sigma_r=.30$, the curve still intersects the diagonal three times, whereas for $\sigma_r=.33$ there is just one point of intersection. Qualitatively, this means that hereabouts there is a bifurcation value σ_r*: as σ_r approaches σ_r* from below, the two stable equilibria contract into the single stable state $x_M(\sigma_r)=.5$. Fig. 4 depicts the graphs of the equilibrium states for the deterministic dynamic as a function of σ_r.

5.4. Steady states of the expected reaction function dynamics

Given the conditional beliefs at some realized state x_t we may form expectations about the next state x_{t+1} which is determined by

$$E[X_{t+1}] = \int_0^1 r_\sigma \circ g_{x\{t\},c}\ (z)\ \mathrm{d}z \tag{6}$$

We define the *expected reaction dynamics* by

$$x_{t+1} = H_{c,\sigma}(x_t) \text{ for } t = 1, 2, \ldots, \tag{7}$$

where $H_{c,\sigma}$:[0,1]→[0,1] is given by

$$H_{c,\sigma}(x) = \int_0^1 r_\sigma \circ\ g_{x,c}(z)\mathrm{d}z \text{ for all } 0 \leq x \leq 1 \tag{8}$$

Thus instead of focusing on single realizations of the process, the function $H_{c,\sigma}$ calculates for each $x \in [0,1]$ the expected percentage of the population willing to join given some informational distribution on the true states. For each σ_r, $H_{c,\sigma}$ represents the "willingness to join" curve

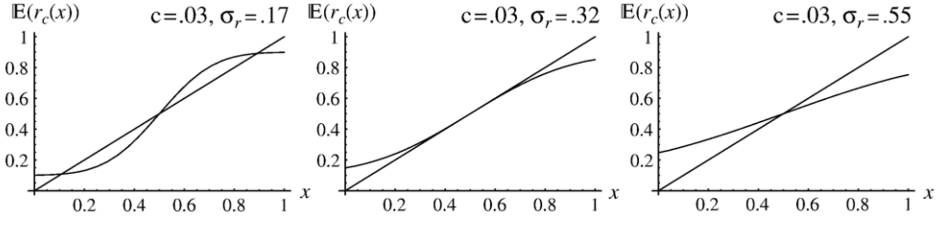

Fig. 5. Expected reaction functions $c=.03$.

depending on the accuracy of the information parametrized by c. Notice that the expected reaction dynamics is deterministic, and the interesting question is as to the properties of the equilibria under formation of expectations given the information component. This amounts to looking at the fixed points of the mapping $H_{c,\sigma}$ and how these fixed points depend on the parameter c. We can do this for different perfect information response curves, parametrized by σ_r.

Fig. 5 shows that initially, for small c, three fixed points are present: one is a repulsor and two are attractors. If the ‘blurriness’ of information increases, however, the dynamics bifurcate to a single attractor which is higher than the original (lowest) attractor: see Figs. 11 and 12.

5.5. Simulations with negligible spread of information, $c=.03$

Recall that the standard deviation for $B(x)$ is at most $c/\sqrt{7}$, approximately 0.011, which would account for good knowledge about the true state. Figs. 13 and 14 show that for the value $\sigma_r=0.17$ the process stays around the lower expected equilibrium state .1. For the intermediate scenario with value $\sigma_r=.32$, where around .5 the graph of r_σ almost coincides with the line $y=x$, we find a clustering of the believed state around .5. Moreover, as can be seen in the left part of the figure, we observe frequent jumps between the two stable states which points at a highly volatile system. Not shown are the results for the value $\sigma_r=.55$, where there is only one stable equilibrium at .5. There is sharp convergence towards this equilibrium state, and the only observed noise is that contained in the assumed information distribution.

5.6. Simulations with low spread of information, $c=.17$

This is the most interesting case, as quite different patterns are observed. For the value $\sigma_r=.17$ the probability of jumping over the state $x=.5$ is still so low that no such occurrence was detected in the first 10^4 iterations. The realized states are centered around the lower-left expected equilibrium value .109. For $\sigma_r=.21$, we have $x_L(\sigma_r)\approx.132$, $x_H(\sigma_r)\approx 1-.132=.868$. The first jump from one side of the middle equilibrium is encountered, followed by a fast convergence towards the upper-right equilibrium state. Nevertheless, jumps are rare. Figs. 15 (relative frequency of states) and 16 (realized states) show that, around $\sigma_r=.26$ the system moves frequently between the two stable equilibria (.198 and .802). These two figures show a similar behavior for values of $\sigma_r\approx.32$, where the system becomes highly volatile. For larger σ_r, with one stable equilibrium left, the noise in the observed states is due to informational assumptions. In Section 5 we will take a closer look at this phenomenon.

5.7. Simulations with moderate spread of information, $c=.32$

As can be observed in Figs. 17 and 18, the probability of misinterpreting the true state is actually so high that the system with three restpoints becomes highly unstable. In the end, with a large spread of the population over the political spectrum the system settles down around the state $x=.5$. In particular this means, in terms of revolution, that anything might happen-anytime.

5.8. Simulations with large spread of information $c>.32$

The system shows complete diffusion over the spectrum of states between the equilibria; for c large enough there is even a non-zero probability that the system needs only one iteration to pass the .5 level. (see Figs. 19 and 20.)

5.9. Obtaining jumping times distributions

Recall the two scenarios (c, σ_r)=(.17,.26) and (c, σ_r)=(.32,.26), whose long-run distributions of states can be considered more or less equal, but whose related state diagrams express a clear-cut difference in volatility. It is in the second scenario, with the larger c, that we find the highest reluctance of the system to jump to another equilibrium state. From the perspective of the revolutionaries it is obviously the more promising of the two scenarios, as it guarantees the best chances for a change of political climate in reasonable time.

We carried out simulations in order to determine the volatility of various scenarios, measured by the duration of the process to reach a support of at least .5. Again, though we could have chosen different criteria, it is this focal point which is in line with Kuran's definition of a revolution.

For each combination (c, σ_r) we performed 10^5 runs; in each we iterated the states $x_1^k, x_2^k, \ldots$ until the first moment $t(k)$ such that $x_{t(k)}>.5$, with a maximum of $t(k)=200$. Fig. 6 shows the relative frequencies of the times $t(k)$ for the runs $k = 1, \ldots, 10^5$, and various scenarios.

The horizontal axis in Figs. 6 and 21 denotes the possible 'revolution times' (first jumping times). Starting with the left equilibrium at $t=1$, the earliest possible revolution time is therefore 2. The vertical axis measures the relative frequencies of the finishing times. The figure shows that with an increasing c, i.e., with increasing blurriness of information, early sparks become more likely. With the low informational level $c=.17$, the relative frequency of any sparking time is almost zero. Not surprisingly, with further increasing values of c, sparks accumulate around the first iteration steps.

For low values of both σ_r and c the dynamics are merely like those of the deterministic system with $f=r_\sigma$, meaning that the system hardly gets out of the lower-left equilibrium state. Interestingly, the influence of the conditional beliefs is increased by a spread of the people over the political spectrum. Just compare the upper-left diagram with the lower-right diagram to notice the sensitivity of the revolting times as functions of the parameters c and σ_r. The scenario $\sigma_r=.32$

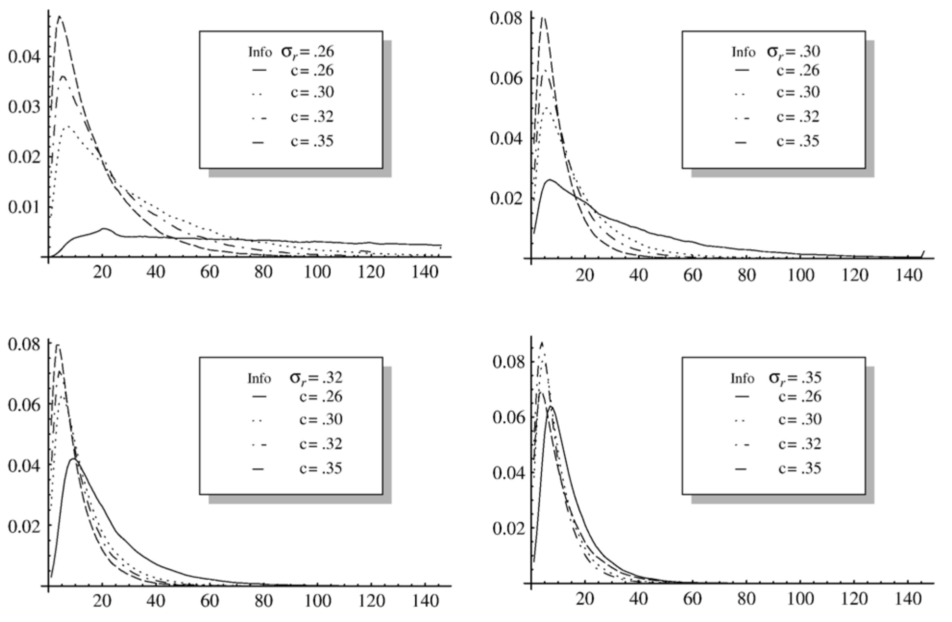

Fig. 6. Revolting times.

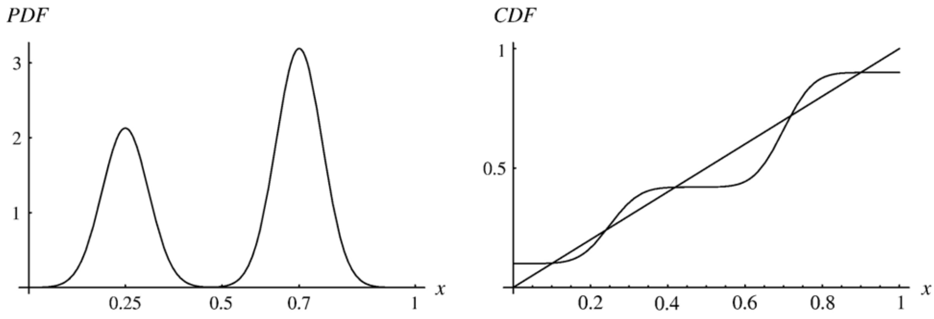

Fig. 7. Polarization and new restpoints at choice of parameters $\lambda=.4$, $c_{\ell}=.25$, $\sigma_{\ell}=.12$, $c_h=.7$, $\sigma_h=.13$.

corresponds to the situation where the people's political preferences are expressed by an almost uniform distribution. It is the case where there is hardly drift towards any of the equilibria, as each state is almost stable for the deterministic dynamics; the sole force underlying the dynamics is the noise in knowledge about the true state. Any further increase in $\sigma_r>.32$ leads to the situation of a single deterministically stable state. Because this equilibrium state is actually .5, even for small values of noise there will be a sharp convergence towards this state.

5.10. *Influencing the reaction curve*

Recall that the reaction curve can be seen as a cumulative distribution function of the mass over the complete political spectrum. The value $r_\sigma(0)$ represents a group of people that will always be on the side of the opposition; private preferences dictate each of these individuals to remain loyal although they might not be able to enjoy the light of revolution. Also on the regime's side one finds the fraction $1-r_\sigma(1)$ of the population who will always be waiting for decay of the opposition, whatever the number of people is that put all their confidence in the adversaries. The *S*-shape reaction curve furthermore indicates that most of the people are concentrated in the middle of the political spectrum, and in particular it shows that most people have no strict preferences. Winning the war means – for both sides – that it is these people that are tilted in their decision. The former section shows that the prospects for a revolutionary movement increase sharply with the diversity of political preference — at least compared to the situation with people centered at .5. Below we will argue that a terrorist action may actually affect the two basics of our dynamics; it will have an impact on the believed state of the world and reshape the threshold function by dictating new aggregate levels of preference. To appreciate the focus on changes in the reaction function, consider a purely deterministic system. In the numerical examples we have shown that small changes of the spread of the political spectrum $\sigma_r\approx.32$ can have enormous impact on the system's equilibrium properties. McCormick and Owen (1996) and Kuran (1989, 1991a,b) each point at diametrically different results, depending on just a small change in kurtosis of the population distribution over the political spectrum[6]. Whoever is capable of influencing this spread has a powerful weapon to invoke the system to adjust in the desired direction. However, although this factor is certainly not to be neglected, we will illustrate that it is probably not the

[6]Kuran suggests that revolutionary actions come in the wake of historic opportunities, while McCormick and Owen feel that historic opportunities can be generated by revolutionary actions.

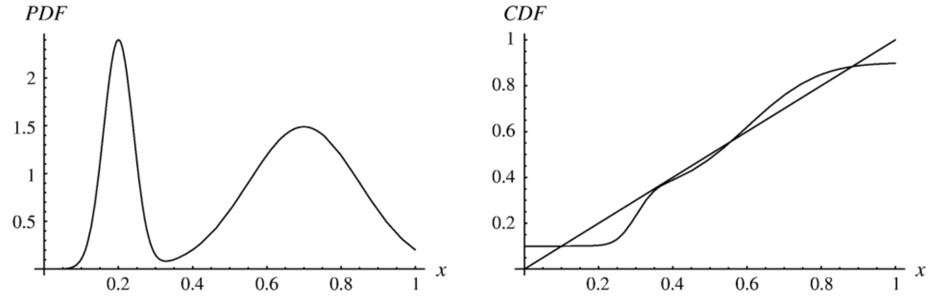

Fig. 8. Sharp polarization: three stable and two unstable equilibria at values of the parameters $\lambda=.3$, $c_\ell=.2$, $\sigma_\ell=.04$, $c_h=.6$, $\sigma_h=.15$.

direct — but the indirect impact of the combination with a certain spread of information that makes the observed kind of attacks worthwhile for insurgent networks. Below we will argue that it is indeed a combination of effects that could resolve a misunderstood part in explaining terrorist actions.

Actions from either side of the political center may induce the people to sharpen their preferences. Recent terrorist attacks by Al Qaida members immediately caused a polarization of the mass into different camps. In Western Europe, the recent bloody assassinations of civilians caused a dramatic shift in sentiment resulting in what were believed to be counter-attacks on Islamic schools. This in turn mobilized part of the Islamic societies to oppose such reactions. This is to say that up to some years ago an S-shaped curve would not misrepresent the actual political spectrum as the aggregate of the different types of populations that lived in peace together. However homogeneous society may have looked from the outside, the latest actions caused shifts within several cultural and ethnic groups. Where nonnegligible groups of Islamic youth are found to radicalize at enormous speed, the cry for a firm and active government indicates a shift in the opposite direction. Then, where an S-shape curve might not fit an accurate realistic dynamic adjustment model, perhaps a double S-shape will. This is illustrated below in Fig. 7 with two peaks in the political spectrum, one in support of the revolutionaries and the other in favor of the government.

In particular, there are now more equilibria, three of which are stable and two unstable. Of interest is this new intermediate equilibrium that is more easily reached for a small revolutionary movement than the upright equilibrium position. In absolute terms, following Kuran's terminology, such action would bring the opposition closer to the desired revolution.

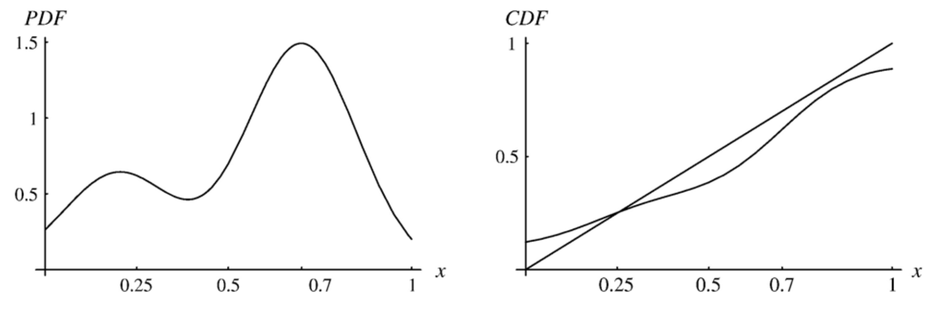

Fig. 9. Overshooting effect: one equilibrium at choice parameters $\lambda=.2$, $c_\ell=.25$, $\sigma_\ell=.12$, $c_h=.6$, $\sigma_h=.13$.

5.11. Layers of political distribution

The discussed distortions of the reaction function can easily be modeled by weighing two types of realization functions: for $\lambda \in (0,1)$ we define

$$r^{\lambda}(x) = \alpha + (1-2\alpha)[\lambda \Phi(x; c_{\ell}\sigma_{\ell})) + (1-\lambda)\Phi(x; c_{h,}\sigma_h)] \tag{9}$$

Such a weighted function may result from aggregating preferences in a heterogeneous population, where cultural as well as social differences may lead to several reaction functions. The weight may be then be interpreted as the mass of the people belonging to a specific subgroup in society. Fig. 7 shows the reaction function r^{λ} for a dichotomous society resulting from mixing the related CDF's given by the normal distributions $N(c_{\ell}\sigma_{\ell}) = N(.25, .12)$ and $N(c_h, \sigma_h) = N(.7, .13)$, respectively, with weight $\lambda = .4$.

The stable equilibrium values are approximately $x_L \approx .12$, $x_M \approx .42$, and $x_H \approx .88$. Thus the value x_M results from an action of the revolutionaries. In between these equilibria we find the unstable ones at .25 and .72.

Now consider the figures below that result from small variations on the above example and note that the number of equilibria and their respective values are highly sensitive to these changes. Fig. 8 may be still be compared with Fig. 7 in a topological sense, as far as the number of equilibria and their respective values are concerned. In Fig. 9 dramatic changes of these characteristics have occurred, as it leaves the opposition in a deterministic dynamical system with only the 'bad' outcome $x_L \approx .26$. Then this will probably not be enough for a revolutionary shift of power — although still this value is higher than the original lower equilibrium value $x_L = .12$. In other words, if the shift to the left is not imminent, the action should have devastating effects for the insurgent network, as the downwards pulling reaction curve would lead to an inferior outcome. Then if the relation between the parameters and the level of action is assumed to be of this sensitive kind, why would terrorists employ attacks?

Though a general answer to this question may not exist, one possible explanation could be the psychological effects of a terrorist attack. First of all, it leaves bewildered many people that would not expect an attack as nearby as was accomplished. It is this (large) group of people that puts confidence in those that are supposed to lead and bring a solution — the governmental institutions. A solution, however, is not achievable, at least not shortly after the action. As a result the reigning power will not get the massive support that one would reckon, which might be just the effect desired by insurgent groups. This leads us to conclude that Fig. 8 seems more adequate as an explanation for the terrorists' rationale than Fig. 9. (Of course the terrorists may be wrong, Fig. 9 might well be the case, and the organization will be badly hurt.)

5.12. Indirect power of information

The shift in mass has another very important indirect effect, and that is that for the new reaction function $\check{r}$ the values $\check{r}(x)$ are closer to x. Figs. 8 and 7 display such equalizing effects due to shifts in sentiment. Earlier discussions already pointed at the fact that an equal spread of the population minimizes the effect of pulling down the believed state to a lower real state of the world. Each point is an approximate restpoint, such that effects of information about the real state of the world gain impact. Propaganda on top of a terrorist action may thus close the gap. This could be just a *lucky shot* strategy — just wait and see what happens: the information gap might close the distance towards a better restpoint.

6. Conclusion

We have presented a dynamic framework by which the mass mobilization problem is discussed in the context of revolutionary movements. In particular the model generalizes other theoretical work by explicitly modeling the beliefs that people might hold about the actual political state of the world as far as their willingness to openly support the revolutionaries is concerned. A major finding is that a lack ofknowledge about the actual level of support is an important source of growth of support for insurgent networks, in particular in the case of a widespread political sentiment. Terrorist actions may serve the revolutionary movement in different ways. First, they increase the range of believed true states as the revolutionary movement signals not only its existence but even its strength. Our examples show that the likeliness of revolution increases with an increase of noise in believed states. Secondly, an important effect is that terror has an equalizing effect on political preferences. Then, the private threshold values being more uniformly distributed, former considerations show an increasing impact of the informational component. In turn, this combination of effects may lead to strong and sudden shifts in sentiment, pulling society over to join the revolutionaries. The effects may be even stronger because the speed at which changes occur may have a more serious impact on the proportion of hardcore membership than suggested here. Any firm basis of support creates a new and increased stable state as status quo, with better sight on the tipping point. In any case, reasons that lead a revolutionary group to victory may as well be the main cause of its failure after installment in power. If the combination of action and lack of information has been the main reason for the jump to another equilibrium, then such phenomena may as well — and just as easily-lead to a jump back. This might explain why revolution often has a follow-up in repression.

Appendix A. Implementation of the dynamics using Mathematica

In order to modify this random generator for our purposes we shall use a transformation technique $Y=h(Z)$, i.e., for a given random variable Z we seek a function h such that Y has a cumulative distribution as prescribed in the last section. The following theorem is taken from Mood et al. (1961), Theorem 11 page 200, tailored to our purposes.

Theorem. Suppose Z is a continuous random variable with values in [0,1] and probability density function $f_Z(\bullet)$. Set $Z=\{z|f_Z(z)>0\}$. Assume that

1. $y=h(z)$ defines a one-to-one transformation $Z\rightarrow[0,1]$,
2. The derivative of $z=h^{-1}(y)$ with respect to y is continuous and nonzero for $y\in[0,1]$, where h^{-1} is the inverse function of h. Then $Y=h(Z)$ is a continuous random variable with density

$$f_y(y) = |\mathrm{d}h^{-1}(y)/\mathrm{d}y| f_z(h^{-1}(y)). \tag{A.1}$$

We first seek the function h such that, when $Z\subset[0,1]$, $Y=h(Z)$ has cumulative distribution function $F_{x,c}$ defined by the anti-derivative of $g_{x,c}$ on $[x-c,x+c]$,

$$F_{x,c}(y) = \int_{x-c}^{y} g_{x,c}(s)\mathrm{d}s \text{ if } x-c\leq y\leq x+c \tag{A.2}$$

Note that in the above theorem we may put $f_Z(z)=1$ for all $z\in[0,1]$, so that Eq. (A.1) becomes

$$f_y(y) = |\mathrm{d}h^{-1}(y)/\mathrm{d}y| \text{ for all } y\in[0,1]. \tag{A.3}$$

For all $y \in (x-c,x+c)$ we can use this equality with

$$f_y(y) = |\mathrm{d}F_{x,s}(y)/\mathrm{d}y| = g_{x,c}(y) \tag{A.4}$$

Assume for the moment that h^{-1} exists and that $\mathrm{d}h^{-1}(y)/\mathrm{d}y$ is positive. Then the function h is the inverse of the function $F_{x,c}$, which exists since $g_{x,c}(y)>0$ for $y \in (x-c,x+c)$. It is easily checked that this function h has all the properties required by in Theorem A.1. Moreover, note that calculating values of h involves finding roots of a polynomial of degree 5, which is easily done in both *Matlab* and *Mathematica*. Thus, we are able to calculate the function

$$h^*(z) = \max\{\min\{h(z), 1\}\}, 0\} \tag{A.5}$$

which has all the properties we need.

Then the pseudocode for one iteration from x_t to x_{t+1} consists of the following steps:

[Step 1] Use Random to generate a random number $z \in [0,1]$ according to the uniform distribution.
[Step 2] Compute $h^*(z)$.
[Step 3] Compute $r(h^*(z))$

Appendix B. Figures

B.1. Simulations

The figures below show the data obtained by 1000 runs of 1000 iteration steps. We counted the relative frequency of states in 10^3 subintervals of type $[a_k,a_k+1]$, with equidistant points $a_1, a_1, \ldots,$ such that $.1 = a_0 < a_1 < \ldots < a_{999} < a_{1000} = .9$.

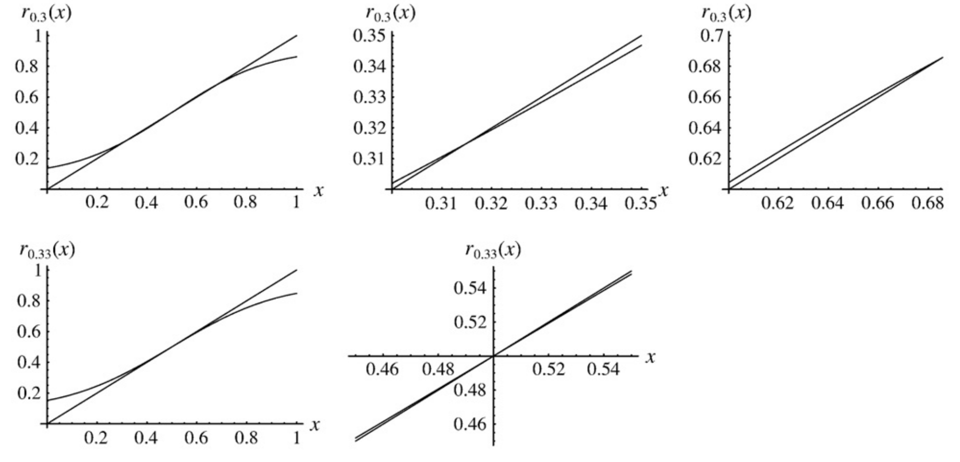

Fig. 10. Bifurcation at $\sigma_r \approx .32$.

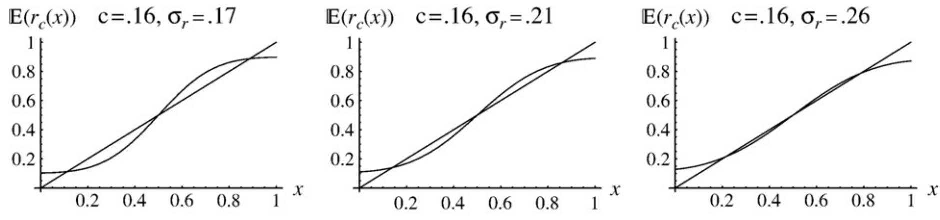

Fig. 11. Expected reaction function $c=.16$.

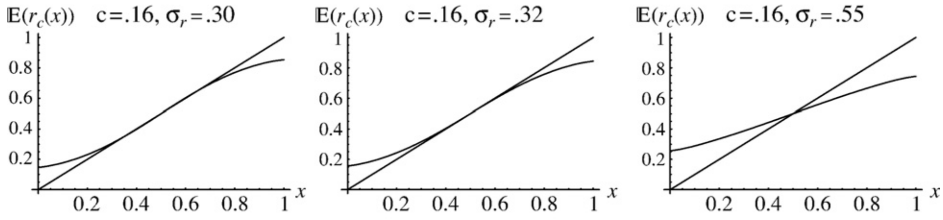

Fig. 12. Expected reaction function $c=.16$.

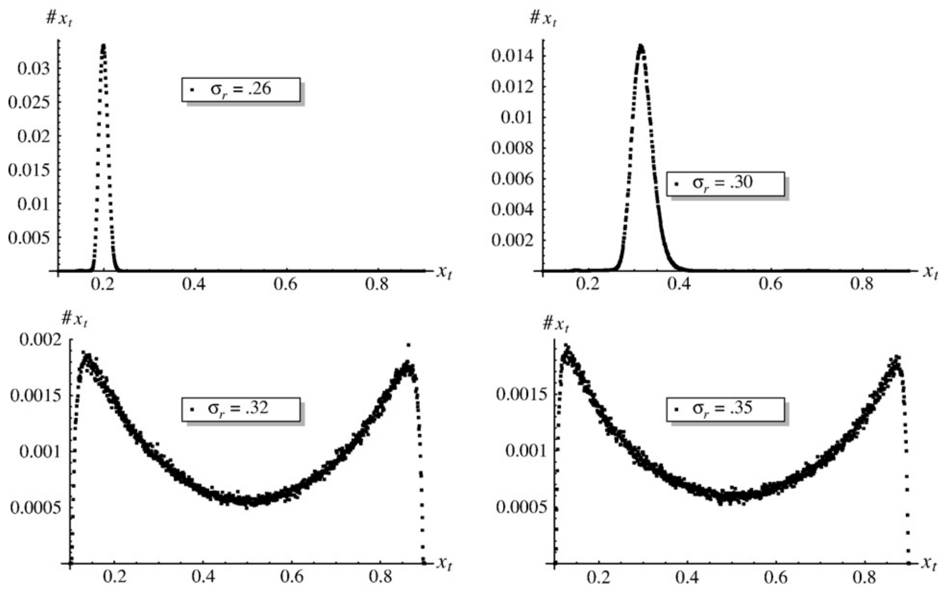

Fig. 13. Relative frequency of states, small spread of information $c=.03$.

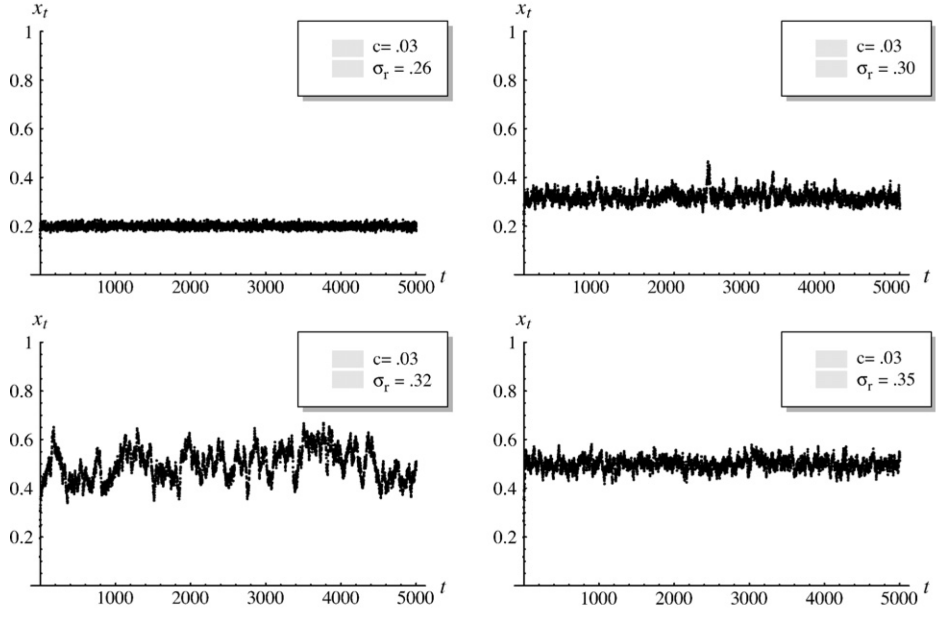

Fig. 14. Realized states, small spread of info c=.03.

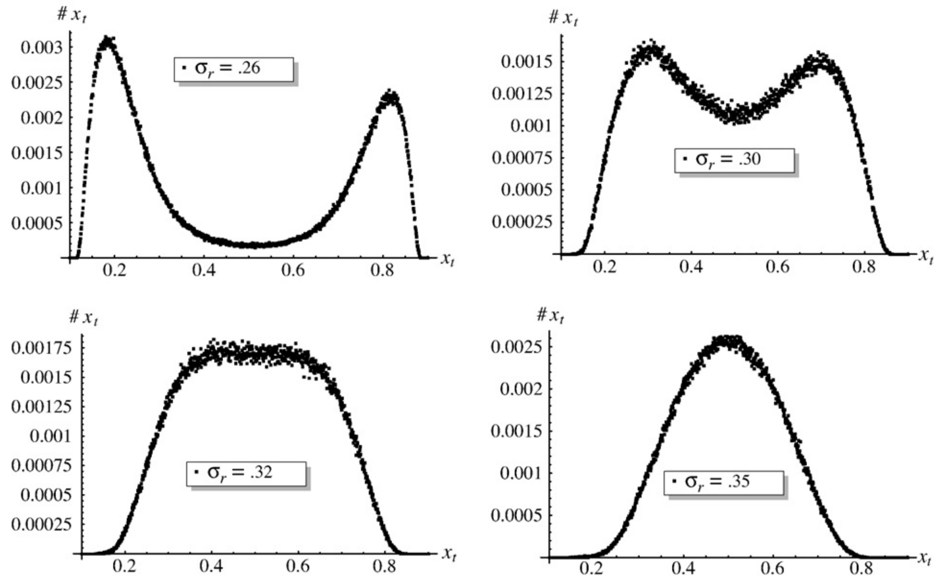

Fig. 15. Relative frequency of states, moderate spread of info c=.17.

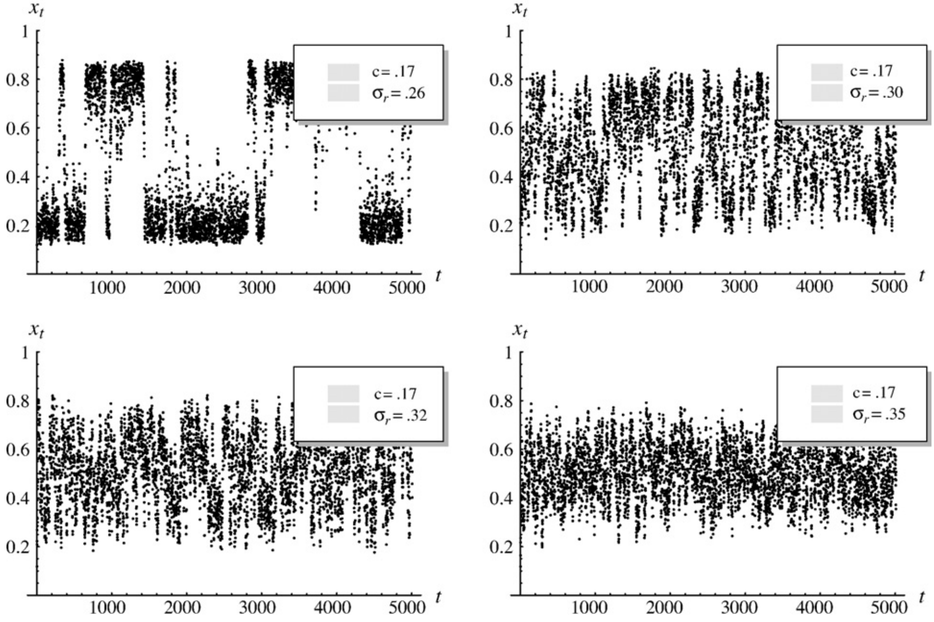

Fig. 16. Realized states, moderate spread of information $c=.17$.

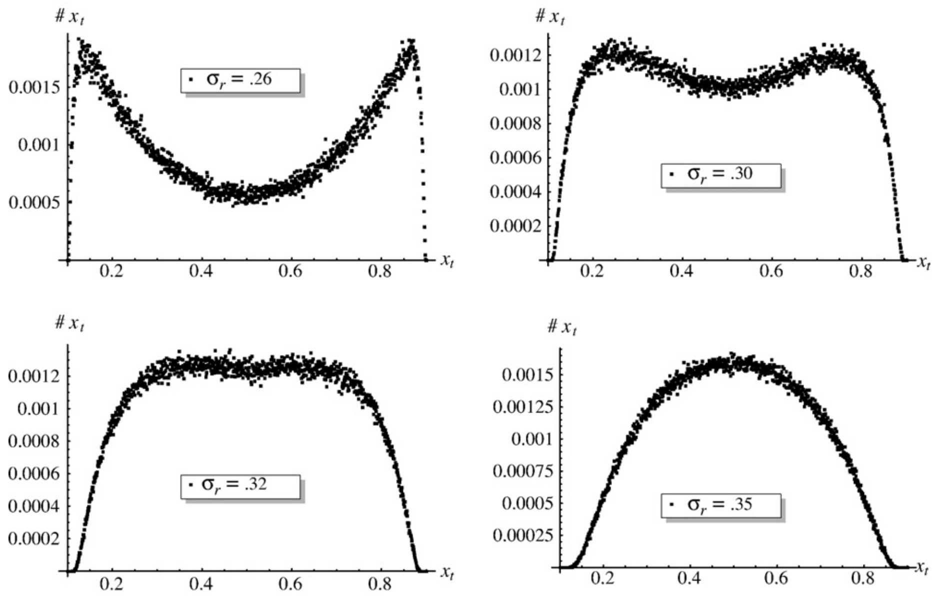

Fig. 17. Relative frequency of states, large spread of info c=.32.

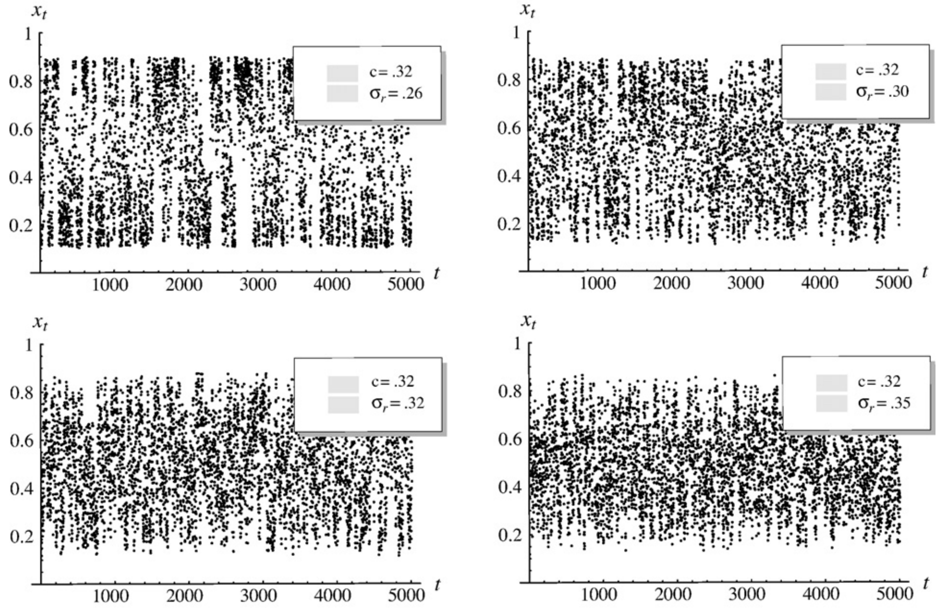

Fig. 18. Realized states, large spread of information c=.32.

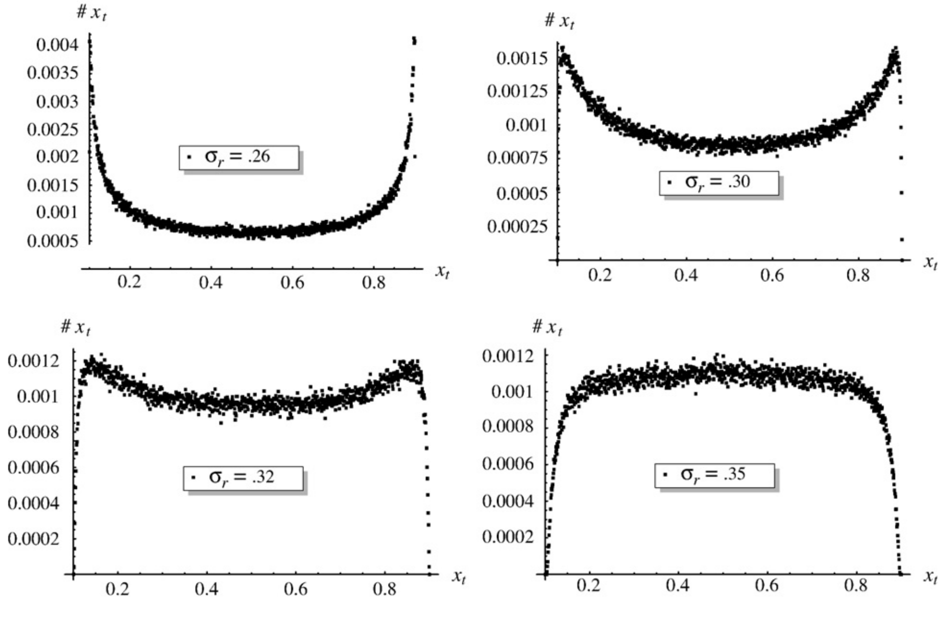

Fig. 19. Relative frequency of states under huge spread of information, c=.55.

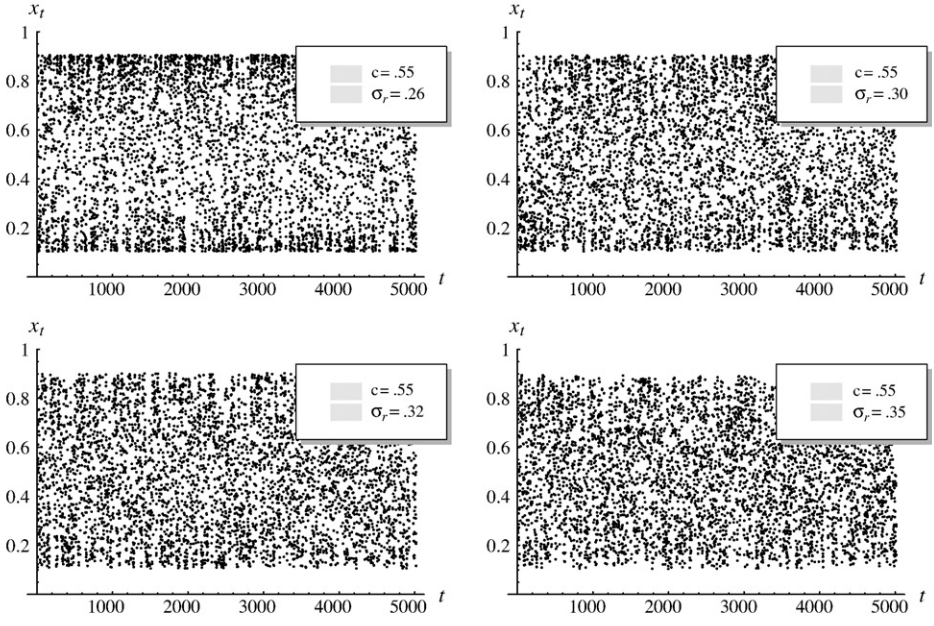

Fig. 20. Realized states under huge spread of information, $c=.55$.

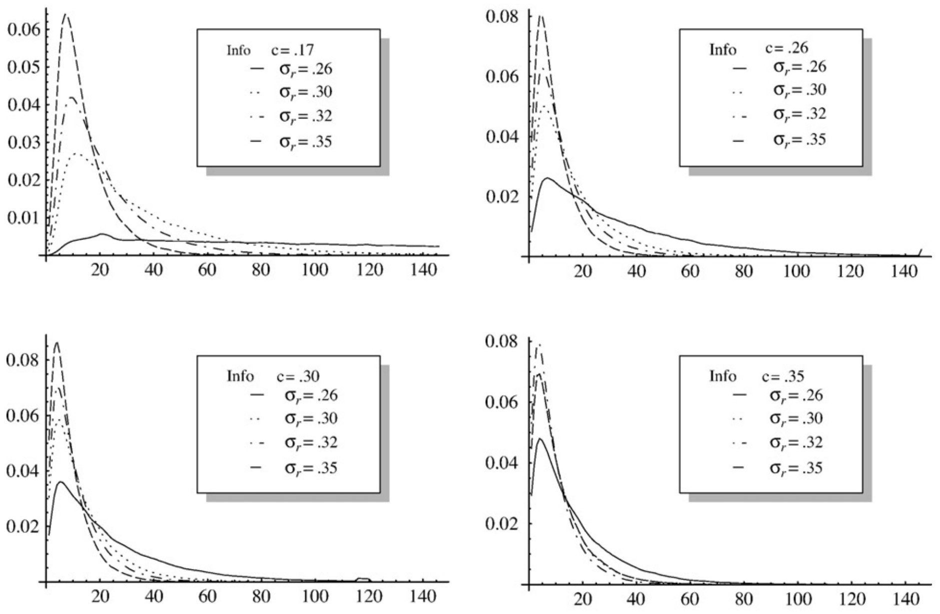

Fig. 21. Revolting times, ordered by beliefs.

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